SIMBa

BGSMath
BARCELONA GRADUATE SCHOOL OF MATHEMATICS

Seminari Informal de Matemàtiques de Barcelona

***Speaker*:** Fabio Ferri.
University: University of Exeter.

Date: Wednesday, March 10th, 2021.
Schedule: 12:00, *virtual coffee break*; 12:20, talk.
Place: Zoom (the link will be posted on our website).
Language: English.

Title: How far is an extension of p-adic fields from having a normal integral basis?

Abstract: Let L/K be a Galois extension of p-adic fields with Galois group G. Denote by $K[G]$ the group ring $\{\sum_{g\in G} a_g g : a_g \in K\}$; the classical normal basis theorem shows that L is a free $K[G]$-module of rank 1, that is, there exists an element $\alpha \in L$ such that $\{g(\alpha)\}_{g\in G}$ is a basis of L as a K-vector space. It is natural to ask whether $\mathcal{O}_L$ is also a free $\mathcal{O}_K[G]$-module of rank 1, where $\mathcal{O}_L$ and $\mathcal{O}_K$ denote the rings of integers of L and K, respectively. A theorem of Noether tells us that this is the case if and only if the extension is (at most) tamely ramified. When L/K is wildly ramified, we can still note that there always exists a free $\mathcal{O}_K[G]$-submodule of $\mathcal{O}_L$ with finite index. The purpose of this talk is to study the minimal such index, i.e. the quantity $m(L/K) := \min_{\alpha\in\mathcal{O}_L}[\mathcal{O}_L : \mathcal{O}_K[G]\alpha]$. We will provide a general bound that only depends on the invariants of the extension, a complete formula for $m(L/K)$ when $L/\mathbb{Q}_p$ is abelian and a complete formula when L/K is cyclic of degree p. This is joint work with Ilaria Del Corso and Davide Lombardo.

About us: *SIMBa* is a youth mathematics seminar organized by graduate students in the Barcelona area. It is aimed towards graduate and last course undergraduate students. Our goals are divulging the knoweledge from different branches of mathematics for those interested and promote networking between the attendants.
This seminar is backed by the Faculty of Mathematics and Computer Science at Universitat de Barcelona, Faculty of Mathematics and Statistics at Universitat Politècnica de Catalunya, the Department of Mathematics from Univesitat Autònoma de Barcelona, CRM, IMUB and BGSMath.

Fore more information, visit at `www.ub.edu/simba/en/`.

If you have any doubt or comment do not hesitate to contact us by sending an email to ***seminari.simba@gmail.com***.