The Lorenz equations Genesis and Generalizations

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Part I

From Navier-Stokes to the Lorenz equations (from the mid-19th century to 1963)



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Introduction

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Introduction

In 1821, the french mathematician *Pierre Simon de Laplace* wrote, in his *Essai philosofique sur les probabilités*:

"We ought then to consider the present state of the universe as the effect of its previous state and as the cause of that which is to follow. An intelligence that, at a given instant, could comprehend all the forces by which nature is animated and the respective situation of the beings that make it up, if moreover it were vast enough to submit these data to analysis, would encompass in the same formula the movements of the greatest bodies of the universe and those of the lightest atoms. For such an intelligence nothing would be uncertain, and the future, like the past, would be open to its eyes..."



Pierre Simon de Laplace Beaumont-en-Auge (Normandia); March, 23, 1749 - Paris; March, 5, 1827

Introduction

Rayleigh-Bénard model Partial differential equations Galerkin's method

Introduction



Jules Henri Poincaré (Nancy, Francia, 29 de April, 29 1854 - Paris, France, 17 de July, 17 1912) About 120 years ago, *Henri Poincaré* in his studies on the stability of the Solar system discovered that its behaviour was, in some sense, chaotic, although the term *chaos* was still several decades away of entering the language of science.

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Introduction

Edward N. Lorenz "Deterministic Nonperiodic Flow" (1963)

• We have the equations that govern the variables that allow the meteorological prediction: Laws of Fluid Mechanics



Edward Norton Lorenz (West Haven, Connecticut, May, 23 1917 -Cambridge, Massachusetts, April, 16 2008)

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- We have supercomputers that place us close to the *extensive intelligence hypothesis* formulated by Laplace...



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Introduction

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 ...and , nevertheless, the weather is unpredictable.



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Introduction

In the present work we have reproduced the path from the fundamental equations of Fluid Mechanics (continuity equation, Navier-Stokes equations and the energy conservation law) to the Lorenz equations. But, furthermore, knowing the process that led *Edward N. Lorenz*, in his studies on convective phenomena at the high layers on the atmosphere, to the discovery, in 1963, of the chaotic dynamics of a three-dimensional system that bears his name, we can pose new consistent problems. On the one hand, in the modification of the approximations made and, on the other hand, in the mathematical study of other more complex possibilities, such as obtaining new systems of decoupled differential equations, with a greater number of state variables.



Introduction

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Introduction

- From now on, we intend to reconstruct the process that guided Lorenz by laying the foundations for the development of these other approaches.
- In addition, in relation with the modification of the approximations made, we have advanced in the process of obtaining the Lorenz type equations in the case in which the coefficient of isothermal compressibility (β) is not null (in the classical context of Lorenz this coefficient was neglected).

Fundamental Laws of Fluid Dynamics

• Law of mass conservation (continuity equation)

$$\partial_t \varrho + \nabla \cdot (\varrho \mathbf{v}) = \mathbf{0} \tag{1}$$



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(2)



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• Law of conservation of energy (heat equation)

$$\partial_t E + \nabla \cdot (E\mathbf{v}) = -\nabla \cdot \mathbf{q} + \nabla \cdot (\mathbf{v}\tilde{\sigma}) + \mathbf{f} \cdot \mathbf{v} + Q$$
(3)

The Batchelor statement for the Rayleigh-Bénard convection problem

A fluid with kinematic viscosity ν , thermal conductivity d_T and volumetric expansion coefficient α fills a rectangular cavity bounded by two isolated vertical walls separated by a distance L and two horizontal walls separated by a height H. If the two horizontal walls are maintained at different temperatures T_1 , on the upper side, and T_2 , on the lower side ($T_1 < T_2$), what is the fluid transfer velocity from one layer to the other?



Simulation of a Rayleigh-Bénard convection fluid



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Rayleigh-Bénard model

To solve this problem it is necessary to make a series of physical hypotheses:

• The velocity is a regular function



Rayleigh-Bénard model

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- Fourier's Law
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- Reaction heat and external heat sources are zero
- The approximation of Oberbeck (1879)-Boussinesq (1901)

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The state law

To close the system of equations it is necessary to introduce a state law, i.e. a relation of the type

 $\varrho = \varrho(T, p)$



The Oberbeck-Boussinesq Approach

• Linear approximation of the state law by means of its Taylor first order expansion

$$\varrho = \varrho_m(1-\alpha(T-T_m)+\beta(p-p_m))$$

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- In this paper, we propose to investigate the possible generalization to the case $\beta \ge 0$.

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PDE's of vorticity and heat conservation

Proposition

The equations of a convective fluid in a rectangular cavity of height H, expressed in terms of the stream function, ψ , and of the deviation from the linear temperature distribution, θ , are, in case where the effects of pressure on density fluctuations are neglected (i.e. $\beta = 0$) the following:

$$\partial_t \nabla^2 \psi + \partial_x \psi \partial_z \nabla^2 \psi - \partial_z \psi \partial_x \nabla^2 \psi - g \alpha \partial_x \theta - \nu \nabla^4 \psi = 0$$

$$\partial_t \theta - \partial_z \psi \partial_x \theta + \partial_x \psi \partial_z \theta - \frac{T_2 - T_1}{H} \partial_x \psi - \kappa \nabla^2 \theta = 0$$



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Nondimensionalization

Proposition

By means of a variable scaling

$$\begin{aligned} x^* &= \frac{x}{H} \\ z^* &= \frac{z}{H} \\ t^* &= \frac{\kappa}{H^2} t \\ \psi^* &= \frac{1}{\kappa} \psi \\ \theta^* &= \frac{g\alpha H^3}{\kappa \nu} \theta \end{aligned}$$

we obtain the following dimensionless equations (continues...)

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Nondimensionalization

Proposition

 $(\dots continuation)$ $\partial_{t^*} \nabla^{*2} \psi^* + \partial_{x^*} \psi^* \partial_{z^*} \nabla^{*2} \psi^* - \partial_{z^*} \psi^* \partial_{x^*} \nabla^{*2} \psi^* - \sigma \partial_{x^*} \theta^* - \sigma \nabla^{*4} \psi^* = 0$ $\partial_{t^*} \theta^* - \partial_{z^*} \psi^* \partial_{x^*} \theta^* + \partial_{x^*} \psi^* \partial_{z^*} \theta^* - Ra \partial_{x^*} \psi^* - \nabla^{*2} \theta^* = 0$ where $\sigma = \frac{\nu}{\kappa}$ es Prandtl number and $Ra = \frac{g \alpha H^3 (T_2 - T_1)}{\kappa}$ is the Rayleigh number.

Fourier Series

In order to find a solution for the PDEs we write the functions ψ^* and θ^* by their Fourier expansion as follows:

$$\psi^{*}(x^{*}, z^{*}, t^{*}) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} [\psi_{mn}(t^{*}) \cos(m\pi ax^{*}) \sin(n\pi z^{*}) + \overline{\psi}_{mn}(t^{*}) \sin(m\pi ax^{*}) \sin(n\pi z^{*}) + \frac{\psi_{mn}(t^{*}) \cos(m\pi ax^{*}) \cos(n\pi z^{*}) + \overline{\psi}_{mn}(t^{*}) \sin(m\pi ax^{*}) \cos(n\pi z^{*})]$$

(and similarly for θ^*). Here we have denoted $a = \frac{H}{L}$ the nondimensional parameter wich we call scale coeficient

Lorenz Equations

Theorem

The Fourier coefficients $\overline{\psi}_{11}(t^*)$, $\theta_{02}(t^*)$ and $\theta_{11}(t^*)$, which are functions of the dimensionless time, t^* , verify the following system of ordinary differential equations:

$$\overline{\psi}_{11}'(t^*) = -\sigma \pi^2 (a^2 + 1) \overline{\psi}_{11}(t^*) + \sigma \frac{a}{\pi (a^2 + 1)} \theta_{11}(t^*)$$

$$\theta_{11}'(t^*) = \pi a R_a \overline{\psi}_{11}(t^*) - \pi^2 (a^2 + 1) \theta_{11}(t^*) + \pi^2 a \overline{\psi}_{11}(t^*) \theta_{02}(t^*)$$

$$\theta_{02}'(t^*) = -4\pi^2 \theta_{02}(t^*) - \frac{1}{2}\pi^2 a \overline{\psi}_{11}(t^*) \theta_{11}(t^*)$$

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Part II

Generalization of the Lorenz equations (2007)



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6 Galerkin's method for the generalized Lorenz model





PDE's of vorticity and heat conservation

Proposition

The equations of a convective fluid in a rectangular cavity of height H, expressed in terms of the current function, ψ , and the deviation from the linear temperature distribution, θ , are the following^a:

$$\partial_{t}\nabla^{2}\psi + \partial_{x}\psi\partial_{z}\nabla^{2}\psi - \partial_{z}\psi\partial_{x}\nabla^{2}\psi - g\alpha\partial_{x}\theta - \nu\nabla^{4}\psi +\beta g \varrho_{m}(\partial_{t}\partial_{z}\psi + \partial_{x}\psi\partial_{zz}\psi - \partial_{z}\psi\partial_{xz}\psi - \nu\partial_{z}\nabla^{2}\psi) = 0 \partial_{t}\theta - \partial_{z}\psi\partial_{x}\theta + \partial_{x}\psi\partial_{z}\theta - \frac{T_{2} - T_{1}}{H}\partial_{x}\psi - \kappa\nabla^{2}\theta = 0$$

^aThese equations are valid for any $\beta \ge 0$.

Complete PDE's Galerkin's method for the generalized Lorenz model

Qualitative study of the generalized model

Nondimensionalization $\beta \geq 0$

Proposition

By scaling variables

$$\begin{aligned} x^* &= \frac{x}{H} \\ z^* &= \frac{z}{H} \\ t^* &= \frac{\kappa}{H^2} t \\ \psi^* &= \frac{1}{\kappa} \psi \\ \theta^* &= \frac{g\alpha H^3}{\kappa \nu} \theta \end{aligned}$$

the following dimensionless equations are obtained (continues...)

Nondimensionalization $\beta \geq 0$

Proposition

(...continuation)

$$\begin{aligned} \partial_{t^*} \nabla^{*2} \psi^* &+ \partial_{x^*} \psi^* \partial_{z^*} \nabla^{*2} \psi^* - \partial_{z^*} \psi^* \partial_{x^*} \nabla^{*2} \psi^* - \sigma \partial_{x^*} \theta^* - \sigma \nabla^{*4} \psi^* \\ &+ \zeta (\partial_{t^*} \partial_{z^*} \psi^* + \partial_{x^*} \psi^* \partial_{z^*z^*} \psi^* - \partial_{z^*} \psi^* \partial_{x^*z^*} \psi^* - \sigma \partial_{z^*} (\nabla^*)^2 \psi^*) = 0 \\ &\partial_{t^*} \theta^* - \partial_{z^*} \psi^* \partial_{x^*} \theta^* + \partial_{x^*} \psi^* \partial_{z^*} \theta^* - R_a \partial_{x^*} \psi^* - \nabla^{*2} \theta^* = 0 \end{aligned}$$

in which the parameters are now, those already mentioned in the work of Lorenz (σ and R_a) and one new additional parameter

$$\zeta = \beta \varrho_m g H$$

which is the coefficient of dimensionless compressibility

Lorenz type equations with $\beta \geq 0$

Theorem

The Fourier coefficients $\overline{\psi}_{11}(t^*)$, $\theta_{11}(t^*)$, $\theta_{02}(t^*)$ y $\overline{\psi}_{11}(t^*)$, which are functions of dimensionless time, t^* , verify the following system of ordinary differential equations:

$$\begin{aligned} \overline{\psi}_{11}'(t^*) &= -\sigma\pi^2(a^2+1)\overline{\psi}_{11}(t^*) + \sigma\frac{a\pi(a^2+1)}{\pi^2(a^2+1)^2+\zeta^2}\theta_{11}(t^*) \\ \theta_{11}'(t^*) &= \pi aR_a\overline{\psi}_{11}(t^*) - \pi^2(a^2+1)\theta_{11}(t^*) + \pi^2 a\overline{\psi}_{11}(t^*)\theta_{02}(t^*) \\ \theta_{02}'(t^*) &= -4\pi^2\theta_{02}(t^*) - \frac{1}{2}\pi^2 a\overline{\psi}_{11}(t^*)\theta_{11}(t^*) \\ \overline{\psi}_{11}'(t^*) &= -\sigma\pi^2(a^2+1)\overline{\psi}_{11}(t^*) + \sigma\frac{a\zeta}{\pi^2(a^2+1)^2+\zeta^2}\theta_{11}(t^*) \end{aligned}$$

Lorenz type equations with $\beta \geq 0$

Corollary

By means of an adequate scaling of time and phase space we obtain a new quadratic system of ordinary differential equations:

$$\begin{aligned} X'(\tau) &= -\sigma X(\tau) + \sigma c^2 Y(\tau) \\ Y'(\tau) &= r X(\tau) - Y(\tau) - X(\tau) Z(\tau) \\ Z'(\tau) &= -b Z(\tau) + X(\tau) Y(\tau) \\ U'(\tau) &= \sigma (1 - c^2) Y(\tau) - \sigma U(\tau) \end{aligned}$$

where $c = rac{\pi(a^2+1)}{\sqrt{\pi^2(a^2+1)^2+\zeta^2}} \in (0,1].$

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Dynamics of the four-dimensional system

Our work concludes with the demonstration that for any value of the parameter $c \in (0, 1]$ the dynamics obtained is the same as that of the classic Lorenz family modulus rescaling of variables and parameters.



Orbits of the four-dimensional system for different values of $c \in (0, 1]$

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Part III

Conclusions













Main contributions

 In our work we have been able to restore all the mathematical process that goes from the Navier-Stokes equations to the Lorenz equations, emphasizing the physical modeling of the problem and the rigorous interpretation of all the variables and parameters that appear in the course.



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- In our work we have been able to restore all the mathematical process that goes from the Navier-Stokes equations to the Lorenz equations, emphasizing the physical modeling of the problem and the rigorous interpretation of all the variables and parameters that appear in the course.
- In addition, it has been possible to submerge all the previous model in a more general one, in which the influence of the pressure in the fluctuations of density of a fluid in convection has been taken into account.

Main contributions

- In our work we have been able to restore all the mathematical process that goes from the Navier-Stokes equations to the Lorenz equations, emphasizing the physical modeling of the problem and the rigorous interpretation of all the variables and parameters that appear in the course.
- In addition, it has been possible to submerge all the previous model in a more general one, in which the influence of the pressure in the fluctuations of density of a fluid in convection has been taken into account.
- Finally, a qualitative study of the generalized model was carried out, concluding that the dynamics of the Lorenz equations in relation to Fluid Mechanics is valid regardless of the degree of influence that the fluid pressure exerts on the density.

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References



Edward N. Lorenz

Deterministic nonperiodic flow

American Meteorological Society, Journal of the Atmospheric Sciencies 20 pp.130-141 Boston, Massachusetts, 1963.



Barry Saltzman

Finite amplitude free convection as an initial value problem-I American Meteorological Society, Journal of the Atmospheric Sciencies 19 pp.329-341 Boston, Massachusetts, 1962.

References



Valentin Joseph Boussinesq *Théorie analytique de la chaleur, vol. 2* Gauthier-Villars Paris, 1903.

Edward A. Spiegel & Georgios Veronis On the Boussinesq approximation for a compressible fluid American Astronomical Society. Astrophysical Journal, vol. 131, pp.442-446 Chicago, 1960.

