Solar Sails: una nova manera de navegar per l'espai

(SIMBa)

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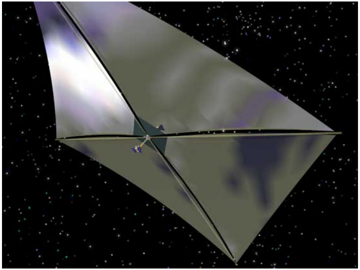
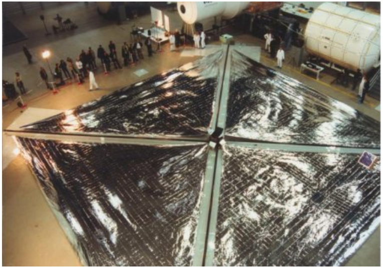
1. Background on Solar Sails
2. Some comments on the natural dynamical properties of the system
3. Station Keeping Strategies Around Equilibria
4. Surfing Along the Family of Equilibria
5. Towards a More Realistic Model
6. Conclusions & Future Work

Background on Solar Sails

What is a Solar Sail ?

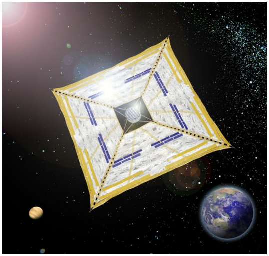
- Solar Sails a proposed form of propulsion system that takes advantage of the Solar radiation pressure to propel a spacecraft.
- The impact of the photons emitted by the Sun on the surface of the sail and its further reflection produce momentum on it.
- Solar Sails open a wide new range of possible missions that are not accessible by a traditional spacecraft.

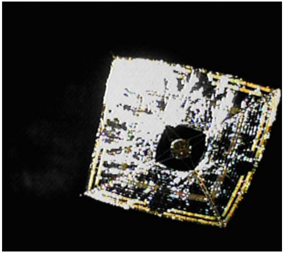




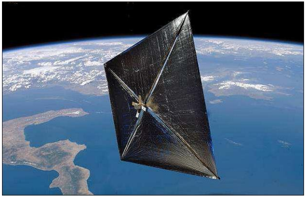
There have recently been two successful deployments of solar sails in space.

- IKAROS: in June 2010, JAXA managed to deploy the first solar sail in space.



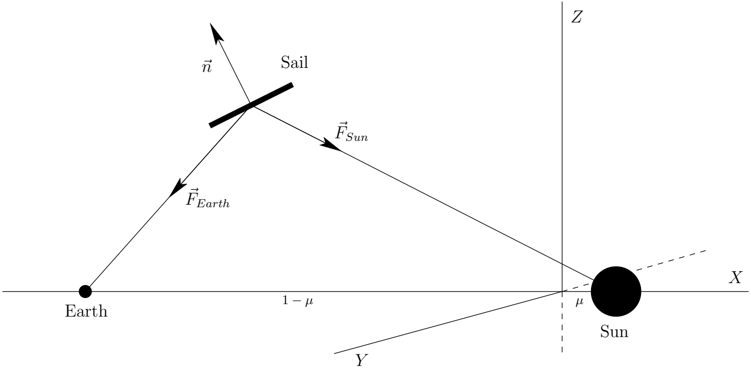


- NanoSail-D2: in January 2011, NASA deployed the first solar sail that would orbit around the Earth.



The Dynamical Model

We use the Restricted Three Body Problem (RTBP) taking the Sun and Earth as primaries and including the solar radiation pressure due to the solar sail.



The Solar Sail

We consider the solar sail to be flat and perfectly reflecting. Hence, the force due to the solar radiation pressure is in the normal direction to the surface of the sail.

The force due to the sail is defined by the *sail's orientation* and the *sail's lightness number*.

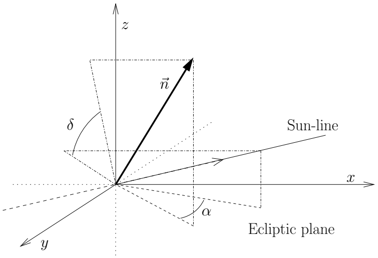
- The *sail's orientation* is given by the normal vector to the surface of the sail, $\vec{n}$. It is parametrised by two angles, α and δ.
- The *sail's lightness number* is given in terms of the dimensionless parameter β. It measures the effectiveness of the sail.

Hence, the force is given by:

$$\vec{F}_{sail} = \beta \frac{m_s}{r_{ps}^2} \langle \vec{r}_s, \vec{n} \rangle^2 \vec{n}.$$

The Sail Orientation

There are several ways to define the two angles that parametrise the sail orientation α and δ.



We define:

- α is the angle between the projection of $\vec{r}_s$ and $\vec{n}$ on the ecliptic plane.
- δ is the difference between: *a*) the angle of the $\vec{r}_s$ with the ecliptic plane; and *b*) the angle of $\vec{n}$ with the ecliptic plane.
- as the sail cannot point towards the Sun, we have that $\langle \vec{r}_s, \vec{n} \rangle \geq 0$.

The Sail Effectiveness

The parameter β is defined as the ratio of the solar radiation pressure in terms of the solar gravitational attraction. It is used to measure the performance of the solar sail. It relates the ratio between the mass of the spacecraft and the size of the sail.

$$\beta = \frac{\sigma^*}{\sigma}, \qquad \sigma^* \approx 1.53 \; g/m^2,$$

where σ is known as the sail loading parameter. With nowadays technology, it is considered reasonable to take $\beta \approx 0.05$, which corresponds to $\sigma \approx 30 \; g/m^2$. For example: a spacecraft of 100 kg needs a square sail of 58 m^2.

Reference: C. McInnes, "Solar Sail: Technology, Dynamics and Mission Applications", Springer-Praxis, 1999.

Equations of Motion

The equations of motion are:

$$\begin{aligned}
\ddot{x} &= 2\dot{y} + x - (1-\mu)\frac{x-\mu}{r_{ps}^3} - \mu\frac{x+1-\mu}{r_{pe}^3} + \beta\frac{1-\mu}{r_{ps}^2}\langle\vec{r}_s,\vec{n}\rangle^2 n_x, \\
\ddot{y} &= -2\dot{x} + y - \left(\frac{1-\mu}{r_{ps}^3} + \frac{\mu}{r_{pe}^3}\right) y + \beta\frac{1-\mu}{r_{ps}^2}\langle\vec{r}_s,\vec{n}\rangle^2 n_y, \\
\ddot{z} &= -\left(\frac{1-\mu}{r_{ps}^3} + \frac{\mu}{r_{pe}^3}\right) z + \beta\frac{1-\mu}{r_{ps}^2}\langle\vec{r}_s,\vec{n}\rangle^2 n_z,
\end{aligned}$$

where $\vec{n} = (n_x, n_y, n_z)$ is the normal direction to the surface of the sail with,

$$\begin{aligned}
n_x &= \cos(\phi(x,y)+\alpha)\cos(\psi(x,y,z)+\delta), \\
n_y &= \sin(\phi(x,y,z)+\alpha)\cos(\psi(x,y,z)+\delta), \\
n_z &= \sin(\psi(x,y,z)+\delta),
\end{aligned}$$

and $\vec{r}_s = (x-\mu, y, z)/r_{ps}$ is the Sun - sail direction.

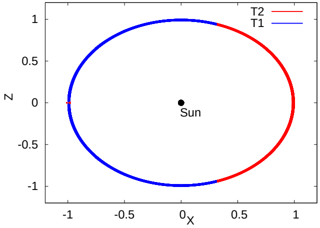
Dynamical Properties of the System

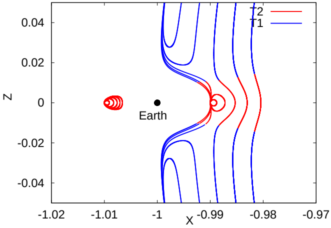
Equilibrium Points (I)

- The RTBP has 5 equilibrium points (L_i). For small β, these 5 points are replaced by 5 continuous families of equilibria, parametrised by α and δ.
- For a fixed small value of β, we have 5 disconnected family of equilibria around the classical L_i.
- For a fixed and larger β, these families merge into each other. We end up having two disconnected surfaces, S_1 and S_2. Where S_1 is like a sphere and S_2 is like a torus around the Sun.
- All these families can be computed numerically by means of a continuation method.

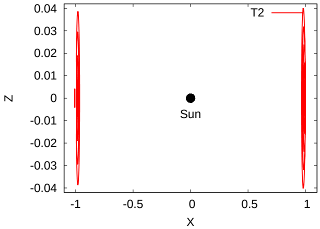
Equilibrium Points (II)

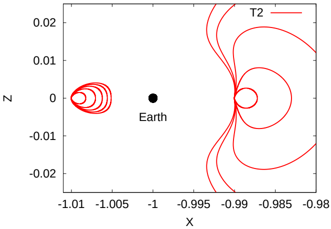
Equilibrium points in the XY plane





Equilibrium points in the XZ plane





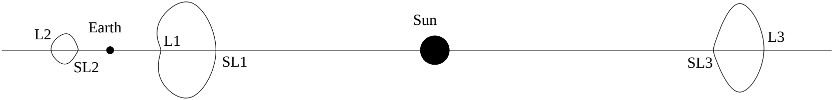
Periodic Motion Around Equilibria

We must add a constrain on the sail orientation to find bounded motion. One can see that when $\alpha = 0$ and $\delta \in [-\pi/2, \pi/2]$ (i.e. only move the sail vertically w.r.t. the Sun - sail line):

- The system is time reversible $\forall\delta$ by $R : (x, y, z, \dot{x}, \dot{y}, \dot{z}, t) \to (x, -y, z, -\dot{x}, \dot{y}, -\dot{z}, -t)$ and Hamiltonian only for $\delta = 0, \pm\pi/2$.
- There are 5 disconnected families of equilibrium points parametrised by δ, we call them $FL_{1,\ldots,5}$ (each one related to one of the Lagrangian points $L_{1,\ldots,5}$).
- Three of these families ($FL_{1,2,3}$) lie on the $Y = 0$ plane, and the linear behaviour around them is of the type saddle$\times$centre$\times$centre.
- The other two families ($FL_{4,5}$) are close to $L_{4,5}$, and the linear behaviour around them is of the type sink$\times$sink$\times$source or sink$\times$source$\times$source.

We focus on ...

- We focus on the motion around the equilibrium on the FL_1 family close to SL_1 (they correspond to $\alpha = 0$ and $\delta \approx 0$).
- We fix $\beta = 0.051689$ (loading parameter $\sigma \approx 30 g/m^2$).
- We consider the sail orientation to be fixed along time.

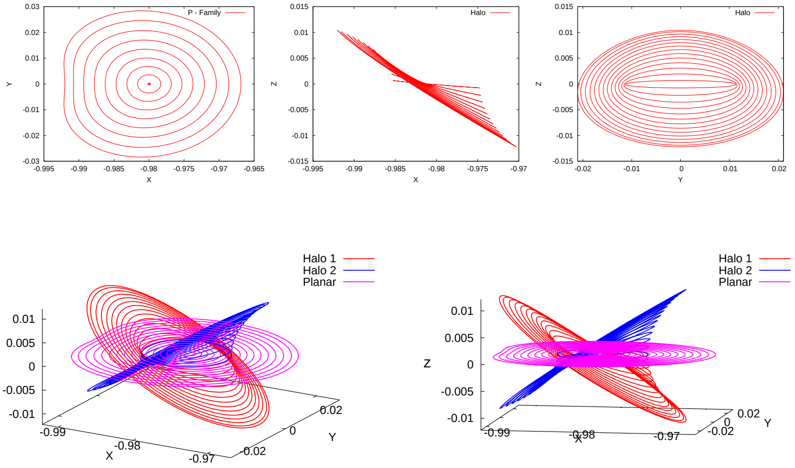


(Schematic representation of the equilibrium points on $Y = 0$)

Let us see the periodic motion around these points for a fixed sail orientation and show how it varies when we change, slightly, the sail orientation.

$\mathcal{P}$-Family of Periodic Orbits

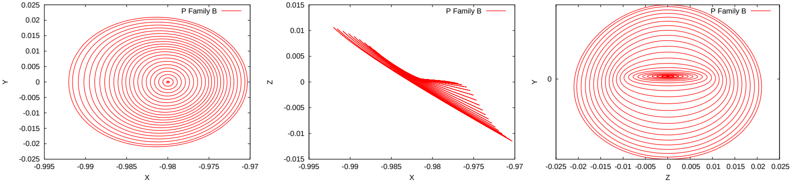
Periodic Orbits for $\delta = 0$.



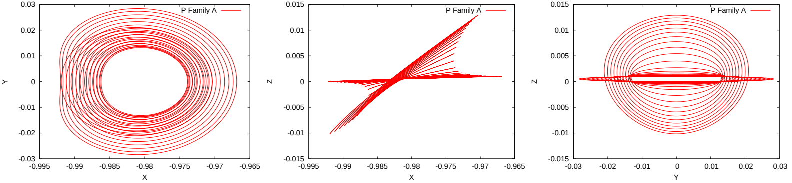
$\mathcal{P}$-Family of Periodic Orbits

Periodic Orbits for $\delta = 0.01$.

Main family of periodic orbits for $\delta = 0.01$

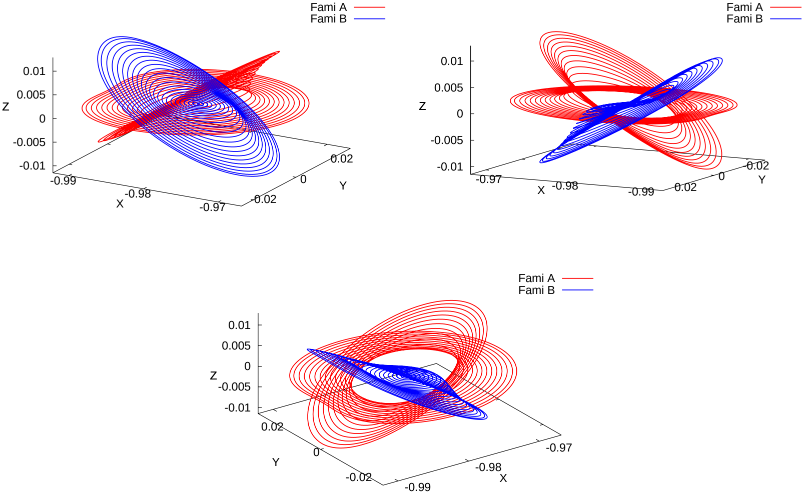


Secondary family of periodic orbits for $\delta = 0.01$

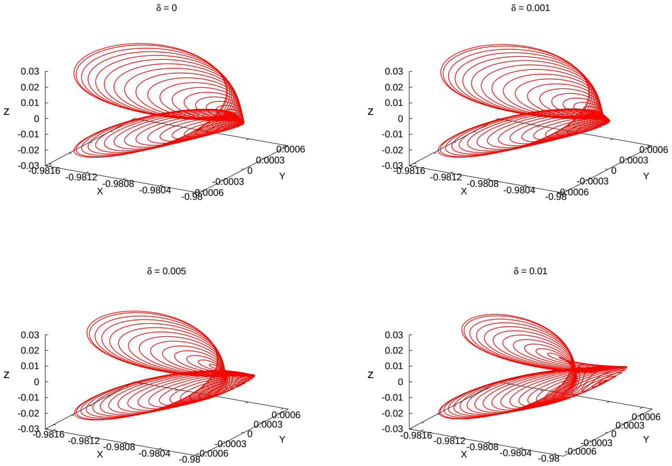


$\mathcal{P}$-Family of Periodic Orbits

Periodic Orbits for $\delta = 0.01$.



$\mathcal{V}$-Family of Periodic Orbits



Reduction to the Centre Manifold

Using an appropriate linear transformation, the equations around the fixed point can be written as,

$$\begin{aligned} \dot{x} &= Ax + f(x,y), \quad x \in \mathbb{R}^4, \\ \dot{y} &= By + g(x,y), \quad y \in \mathbb{R}^2, \end{aligned}$$

where A is an elliptic matrix and B an hyperbolic one, and $f(0,0) = g(0,0) = 0$ and $Df(0,0) = Dg(0,0) = 0$.

- We want to obtain $y = v(x)$, with $v(0) = 0$, $Dv(0) = 0$, the local expression of the centre manifold.
- The flow restricted to the invariant manifold is

$$\dot{x} = Ax + f(x, v(x)).$$

On the Centre Manifold

We have computed the centre manifold around different equilibrium points of the FL_1 family up to degree 16.

- After this reduction we are in a four dimensional phase space (x_1, x_2, x_3, x_4) that is difficult to visualise.
- We need to perform suitable Poincaré sections to reduce the phase space dimension and help us visualise the phase space.
- For $\delta = 0$ the system is Hamiltonian and we have a first integral. We will use it to reduce the phase space dimension.
- For $\delta \neq 0$ the system is no longer Hamiltonian. We will take a function that varies little along the trajectories as an "approximate first integral", and use it to visualise the phase space.

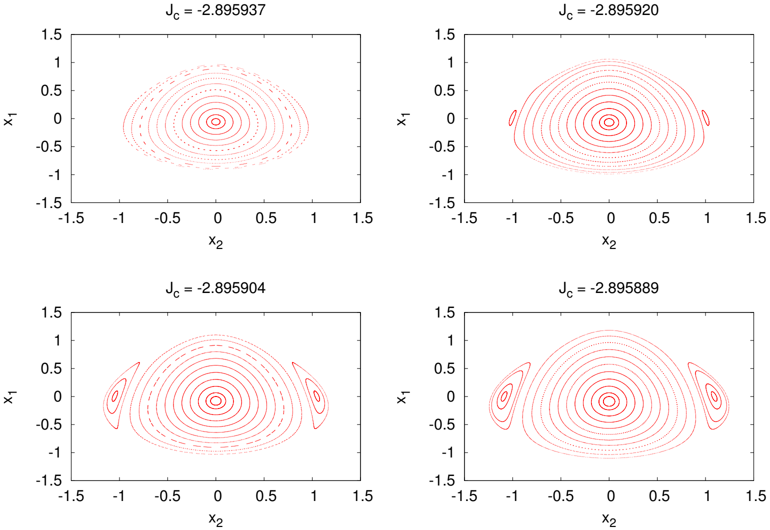
Dynamics for $\delta = 0$

Here the first integral is:

$$J_c = (\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2) - 2\Omega(X, Y, Z).$$

- We fix a Poincaré section $x_3 = 0$ to reduce the system to a three dimensional phase space. *(Taking $x_3 = 0$ is like taking $Z = 0$)*.
- We fix the energy level to determine x_4 and reduce the system to a two dimensional phase space that is easy to visualise. *(Taking $x_4(J_c, x)$ is like taking $\dot{Z}(J_c, x)$)*.
- We have taken several initial conditions and computed their successive images on the Poincaré section.

Dynamics for $\delta = 0$ $(x_3 = 0$ *section)*



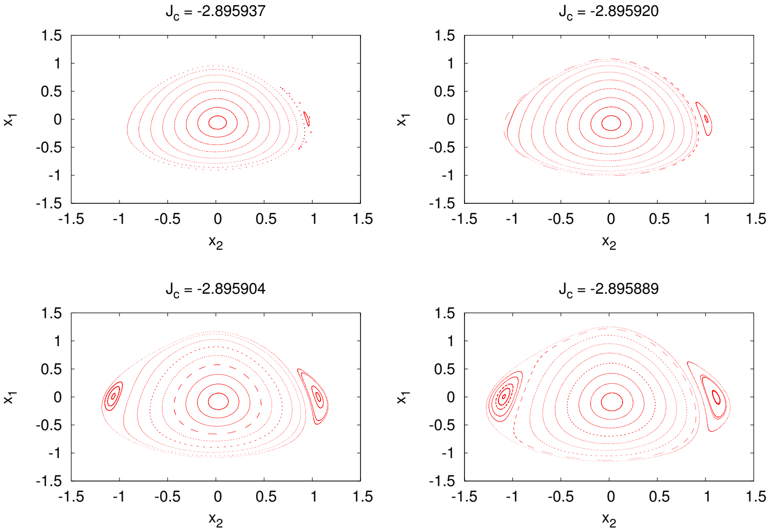
Dynamics for $\delta \neq 0$

Here we take an *"approximated first integral"* :

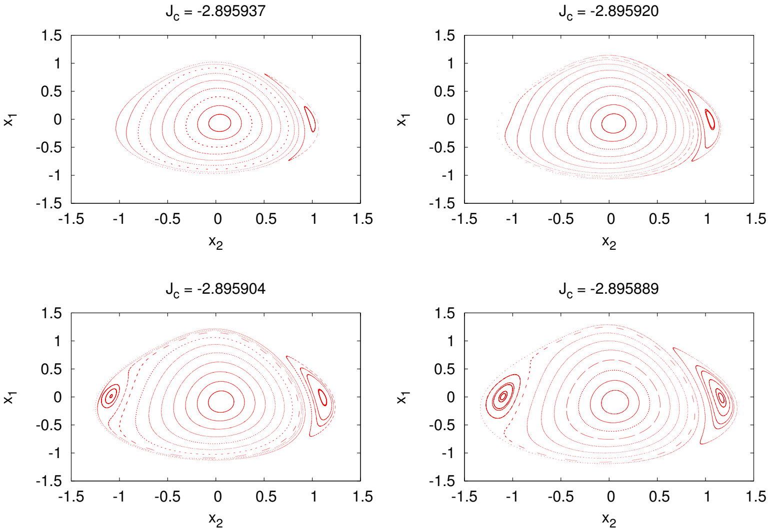
$$J_c = (\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2) - 2\Omega(X, Y, Z) + \beta(1-\mu)\frac{Zr_2}{r_{PS}^3}\cos^2\delta\sin\delta$$

- We fix a Poincaré section $x_3 = 0$ to reduce the system to a three dimensional phase space. *(Taking $x_3 = 0$ is similar to taking $Z = Z^*$).*
- We fix J_c to determine x_4 and reduce the system to a two dimensional phase space that is easy to visualise. *(Taking $x_4(J_c, x)$ is like taking $\dot{Z}(J_c, x)$).*
- We have taken several initial conditions and computed their successive images on the Poincaré section.

Dynamics for $\delta = 0.005$ $(x_3 = 0$ section$)$



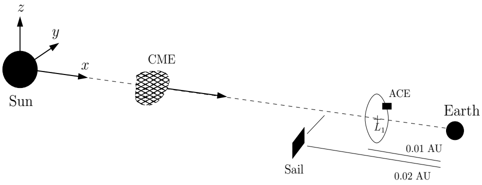
Dynamics for $\delta = 0.01$ ($x_3 = 0$ section)



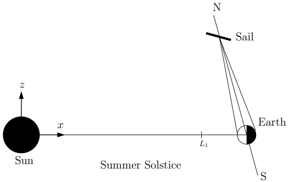
Station Keeping Strategies Around Equilibria

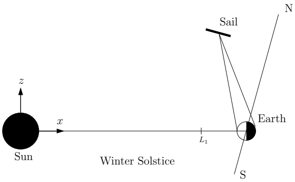
Interesting Missions Applications

Observations of the Sun provide information of the geomagnetic storms, as in the Geostorm Warning Mission.



Observations of the Earth's poles, as in the Polar Observer.





Station Keeping for a Solar Sail

We want to design station keeping strategy to maintain the trajectory of a solar sail close to an unstable equilibrium point.

Instead of using *Control Theory Algorithms*, we want to use *Dynamical System Tools* to find a station keeping algorithm for a Solar Sail.

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Instead of using *Control Theory Algorithms*, we want to use *Dynamical System Tools* to find a station keeping algorithm for a Solar Sail.

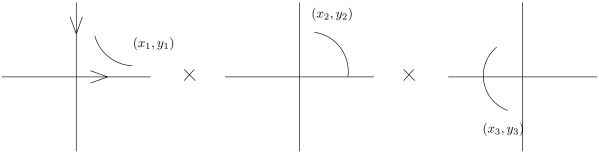
The main ideas are ...

- To focus on the linear dynamics around an equilibrium point and study how this one varies when the sail orientation is changed.
- To change the sail orientation (i.e. the phase space) to make the system act in our favour: keep the trajectory close to a given equilibrium point.

Station Keeping for a Solar Sail

We focus on the two previous missions, where the equilibrium points are unstable with two real eigenvalues, $\lambda_1 > 0, \lambda_2 < 0$, and two pair of complex eigenvalues, $\nu_{1,2} \pm \mathrm{i}\omega_{1,2}$, with $|\nu_{1,2}| << |\lambda_{1,2}|$.

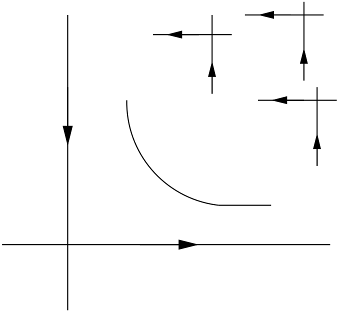
- To start we can consider that the dynamics close the equilibrium point is of the type saddle $\times$ centre $\times$ centre.
- From now on we describe the trajectory of the sail in three reference planes defined by each of the eigendirections.



- For small variations of the sail orientation, the equilibrium point, eigenvalues and eigendirections have a small variation. We will describe the effects of the changes on the sail orientation on each of these three reference planes.

Schematic Idea of the Station Keeping Strategy (I)

In the saddle projection of the trajectory:

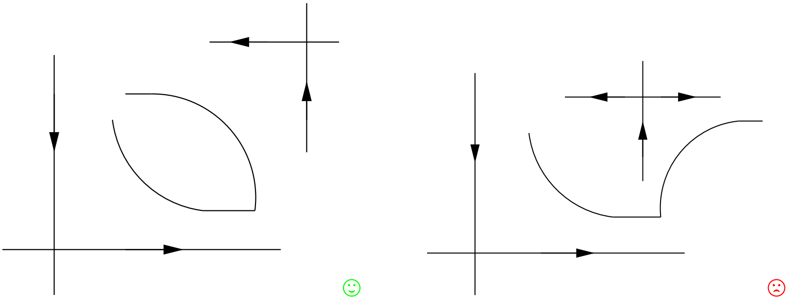


- When we are close to the equilibrium point, p_0, the trajectory escapes along the unstable direction.
- When we change the sail orientation the position of the equilibrium point is shifted and its eigendirections vary slightly.

Schematic Idea of the Station Keeping Strategy (II)

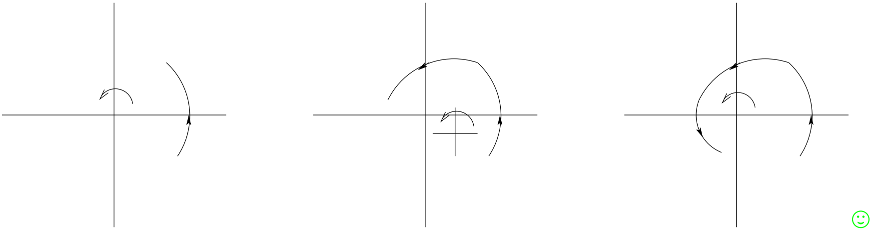
In the saddle projection of the trajectory:

- Now the trajectory will escape along the new unstable direction.
- We want to find a new sail orientation (α, δ) so that the trajectory will come close to the stable direction of p_0.



Schematic Idea of the Station Keeping Strategy (III)

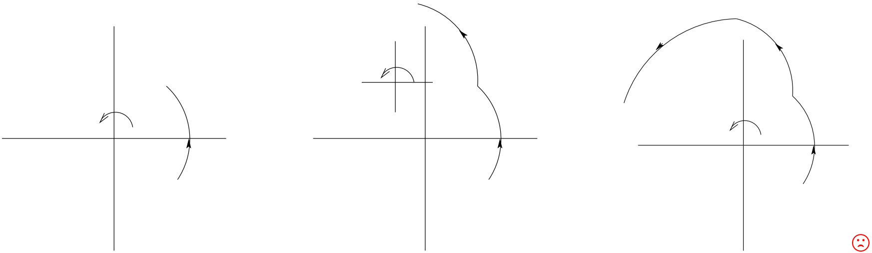
In the centre projection of the trajectory:



A sequence of changes on the sail orientation implies a sequence of rotations around different equilibrium points on the centre projection, which can result of an unbounded growth.

Schematic Idea of the Station Keeping Strategy (III)

In the centre projection of the trajectory:



A sequence of changes on the sail orientation implies a sequence of rotations around different equilibrium points on the centre projection, which can result of an unbounded growth.

Schematic idea of the Station Keeping Algorithm

We look at the sails trajectory in the reference system $\{x_0; \vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4, \vec{v}_5, \vec{v}_6\}$, so $z(t) = x_0 + \Sigma_i s_i(t)\vec{v}_i$.

During the station keeping algorithm:

1. when $\alpha = \alpha_0, \delta = \delta_0$: if $|s_1(t)| \geq \varepsilon_{max}$ $\Rightarrow$ choose new sail orientation $\alpha = \alpha_1, \delta = \delta_1$.
2. when $\alpha = \alpha_1, \delta = \delta_1$: if $|s_1(t)| \leq \varepsilon_{min}$ $\Rightarrow$ restore the sail orientation: $\alpha = \alpha_0, \delta = \delta_0$.
3. Go Back to 1.

1st Idea for finding $\alpha_{new}, \delta_{new}$

We will choose a the position of the new equilibrium point (i.e. a new sail orientation) so that projection of the trajectory on the saddle will come back and the two centre projections remain bounded ?

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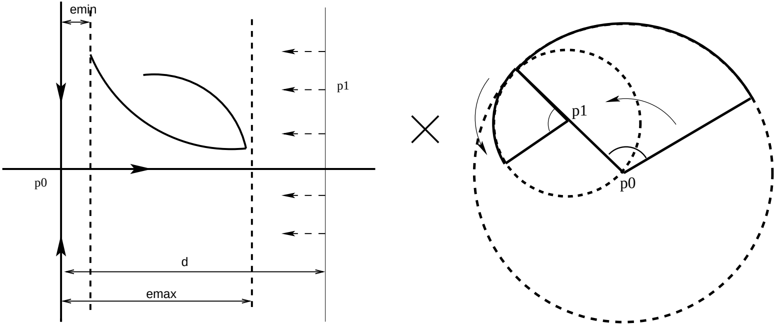


The constants ε_{min}, ε_{max} and d will depend on the mission requirements and the dynamics around the equilibrium point.

Remarks

- We do not know explicitly the position of the equilibrium points $p(\alpha, \delta)$. But we can compute the linear approximation of this function:

$$p(\alpha, \delta) = p(\alpha_0, \delta_0) + Dp(\alpha_0, \delta_0) \cdot (\alpha - \alpha_0, \delta - \delta_0)^T.$$

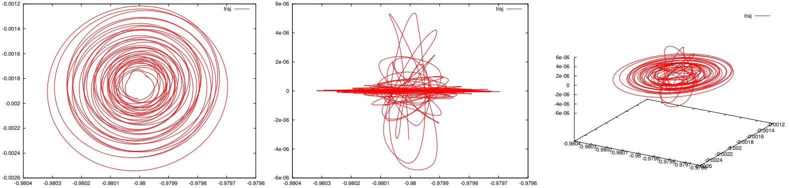
- There are some restrictions of the position of the new equilibria when we change α and δ. We have 2 unknowns and at least 6 conditions that must be satisfied.

- We will change the sail orientation so that the position of the new fixed point is as close as possible to the desired new equilibrium point and in the correct side in the saddle projection.

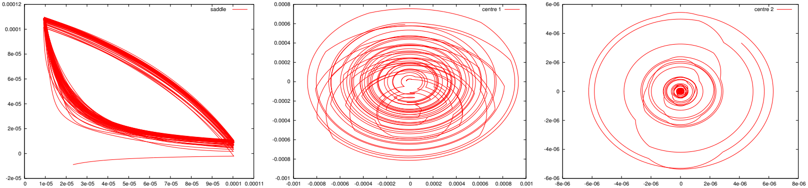
- To decide the new sail orientation we will assume that the eigenvalues and eigendirections do not vary when the sail orientation is changed.

Results for the Geostorm Mission (RTBPS)

XY and XZ and XYZ Projections

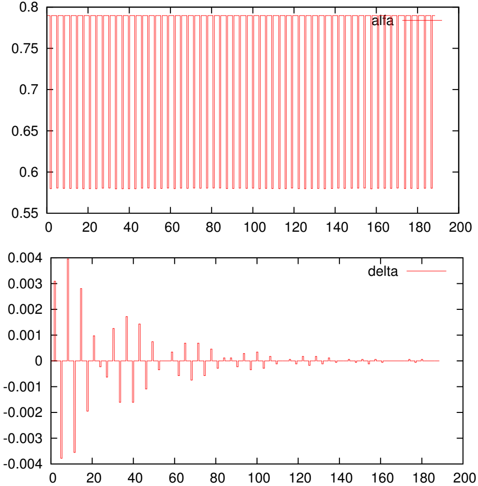


Saddle × Centre × Centre Projections



Results for the Geostorm Mission (RTBPS)

Variation of the sail orientation



2st Idea for finding $\alpha_{new}, \delta_{new}$

The computation of variational equations (of suitable order) w.r.t. α and δ gives explicit expressions for the effect of different orientations (close to the reference values $\alpha = \alpha_0, \delta = \delta_0$) trajectory.

$$\phi_t(x_0, \alpha_0 + h_a, \delta_0 + h_d) = \phi_t(x_0, \alpha_0, \delta_0) + \frac{\partial \phi}{\partial \alpha}(x_0, \alpha_0, \delta_0) \cdot h_a + \frac{\partial \phi}{\partial \delta}(x_0, \alpha_0, \delta_0) \cdot h_d,$$

With this we can impose conditions on the “future” of the orbit and find orientations that fulfil them (or show that the condition is unattainable).

- We will define the parameters ε_{max}, Dt_{min} and Dt_{max} that will vary for each mission application.
- We will find $\alpha_{new}, \delta_{new}$ and $dt \in [Dt_{min}, Dt_{max}]$ so that the trajectory is close to the fixed point.

Remarks

We use the variational equations up to first order. Hence, we have a linear map for the different final states.

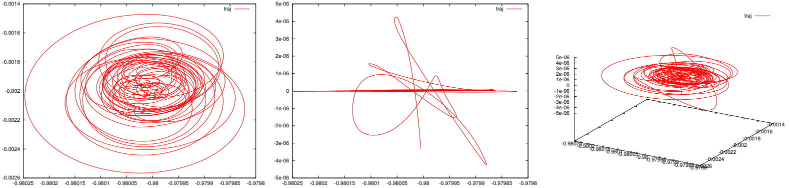
As before we want the final position to be close to the stable direction, keeping small the two centre projections.

One can think of different ways to solve this problem. We have seen that the best results are found if we:

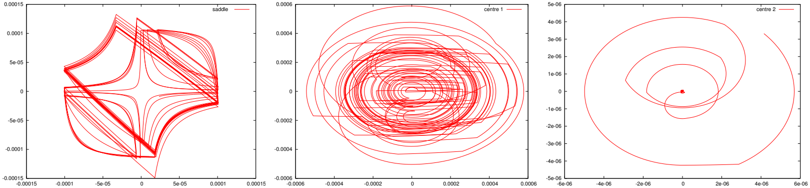
- For each $dt \in [Dt_{min}, Dt_{max}]$ we will find α_{new} and δ_{new} such that $s_1 = 0$ and (s_5, s_6) are minimum (i.e. we are close to stable direction and one of the centres is small).
- We finally choose the dt, α_{new} and δ_{new} that minimises the other centre projection (s_3, s_4).

Results for the Geostorm Mission (RTBPS)

XY and XZ and XYZ Projections

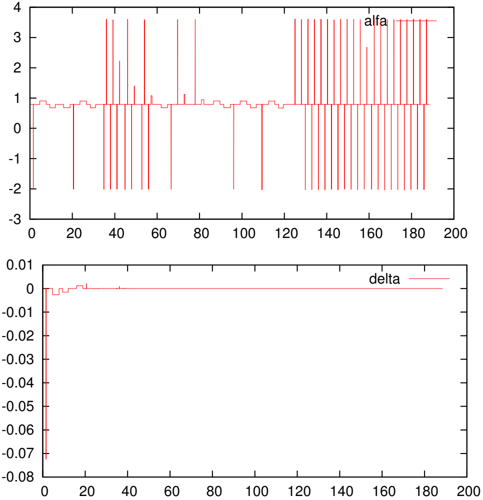


Saddle × Centre × Centre Projections



Results for the Geostorm Mission (RTBPS)

Variation of the sail orientation



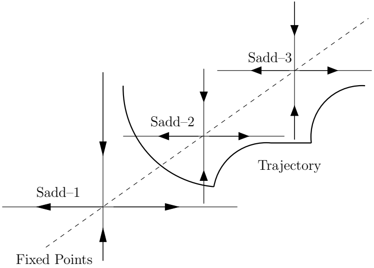
Surfing Along the Family of Equilibria

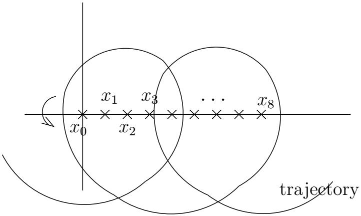
Surfing along the family of equilibria

- Using the same ideas, we can derive strategies to drift along the families of equilibria in a controlled way (use the invariant manifolds to move in the phase space).
- Lets assume that we are close to an equilibrium point p_0 and we want to reach the vicinity of another equilibrium point p_f. Once we reach p_f we want to be able to remain there for a long time.
- We want to find a sequence of changes on the sail orientation (α_i, δ_i) (i.e. a sequence of fixed points p_i) so that the sequence of stable/unstable directions of p_i guide the probe to the final point.
- Note that we also need to take into account the projection of the trajectory on the centre component.

Surfing along the family of equilibria

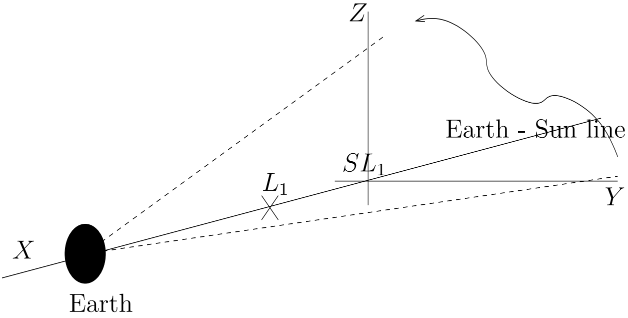
Scheme on the idea to surf along the family of equilibria.





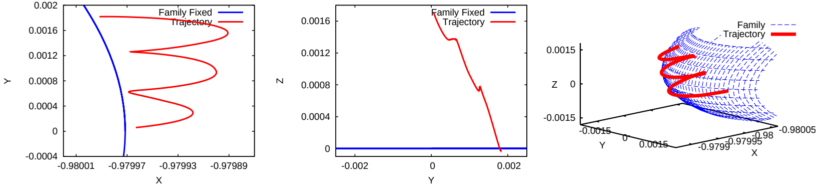
Proposed Missions / Toy Missions

We propose a toy mission to surf along the family of equilibria. We take the Geostorm mission scenario and we want to change the position of the solar sail.

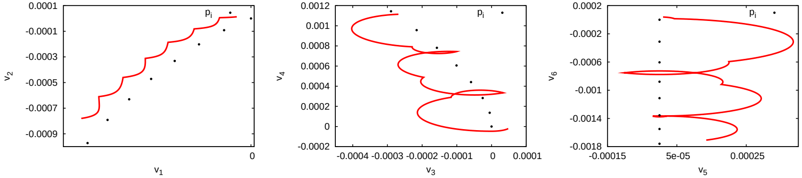


Surfing to gain vertical displacement

XY and XZ and XYZ Projections

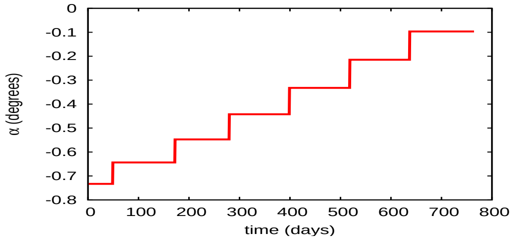


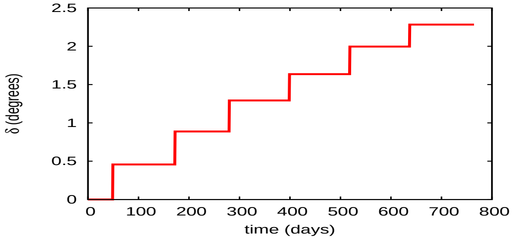
Saddle × Centre × Centre Projections



Surfing to gain vertical displacement

Variation of the sail orientation along time.





Towards a More Realistic Model

Including more realism to the dynamical model

There are several ways to include more realism to the dynamical model. For example,

- taking a more realistic model for the Solar Sail by including the force produced by the absorption of the photons, the reflectivity properties of the sail material, ...
- taking a more realistic model for the gravitational perturbations by including the eccentricity in the Earth - Sun system. Or the gravitational attraction of other bodies, i.e. the Moon, Jupiter, ...

We have started by considering the eccentricity in the Earth - Sun system and studied the robustness of our strategies. So we take the Elliptic Restricted Three Body Problem with a Solar sail as a model.

The Dynamical Model

We use a Restricted N-Body Problem taking the Sun, Earth, Jupiter and solar sail, including the solar radiation pressure due to the solar sail.

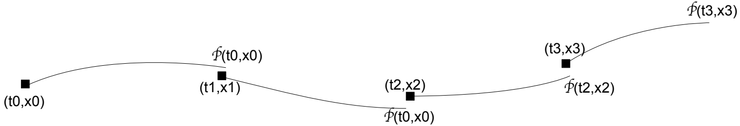
The equations of motion are,

$$\begin{aligned}
\ddot{x}_s &= \sum_{i=0}^{n} Gm_i \frac{x_i - x_s}{r_{is}^3} + \beta \frac{Gm_0}{r_{0s}^2} \langle \vec{r}_s, \vec{n} \rangle^2 n_x, & \ddot{x}_i &= \sum_{j \neq i} Gm_j \frac{x_i - x_j}{r_{ij}^3}, \\
\ddot{y}_s &= \sum_{i=0}^{n} Gm_i \frac{y_i - y_s}{r_{is}^3} + \beta \frac{Gm_0}{r_{0s}^2} \langle \vec{r}_s, \vec{n} \rangle^2 n_y, & \ddot{y}_i &= \sum_{j \neq i} Gm_j \frac{y_i - y_j}{r_{ij}^3}, \\
\ddot{z}_s &= \sum_{i=0}^{n} Gm_i \frac{z_i - z_s}{r_{is}^3} + \beta \frac{Gm_0}{r_{0s}^2} \langle \vec{r}_s, \vec{n} \rangle^2 n_z, & \ddot{z}_i &= \sum_{j \neq i} Gm_j \frac{z_i - z_j}{r_{ij}^3},
\end{aligned}$$

where (x_s, y_s, z_s) and (x_i, y_i, z_i) are the position of the solar sail and the planets respectively, where $i = 0, ..., n$ stands for the Sun, and the other planets.

Nominal Orbit

To find a good nominal orbit we have implemented a parallel shooting method to get a natural trajectory in the Sun - Earth - Jupiter model close to the fixed point in the RTBP Sun - Earth model.



- The points x_i that belong to the nominal orbit must satisfy:

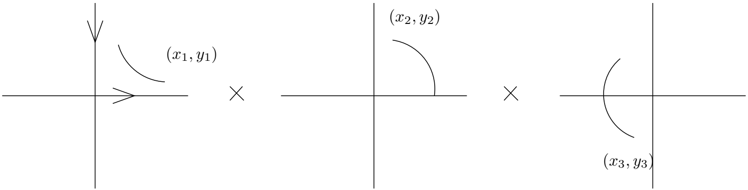
$$\phi_\tau(t_i, x_i) = x_{i+1} \quad \text{for } i = 0, \dots, n-1.$$

- This leads to solving a non-linear equation with $6n$ equations and $6n + 6$ unknowns.
- We have added six more conditions: we fix the initial positions (the first three components of x_0) and the final ones (the first three components of x_n).

Linear Dynamics

If we examine the eigenvalues of the variational flow along the nominal orbit we have: two real eigenvalues, $\lambda_1 > 0, \lambda_2 < 0$, and two pair of complex eigenvalues, $\nu_{1,2} \pm \mathrm{i}\omega_{1,2}$, with $|\nu_{1,2}| << |\lambda_{1,2}|$.

- A first good approximation for the linear dynamics along the nominal orbit is: saddle × centre × centre.



- To describe the trajectory of the sail along the orbit we will use the three reference planes defined by the corresponding eigenvectors.

Reference Frame

To avoid numerical problems with the computation of the eigenvectors, we split the nominal orbit into N revolutions (each revolution = 1 year). For each revolutions we can compute the Floquet modes and use them as a reference frame along the orbit.

$$\begin{aligned}
\vec{v}_1(t) &= e_1(\tau)\exp\left(-\tfrac{\tau}{T}\ln\lambda_1\right), \\
\vec{v}_2(t) &= e_2(\tau)\exp\left(-\tfrac{\tau}{T}\ln\lambda_2\right), \\
\vec{v}_3(t) &= \left[\cos\left(-\Gamma_1\tfrac{\tau}{T}\right)e_3(\tau)-\sin\left(-\Gamma_1\tfrac{\tau}{T}\right)e_4(\tau)\right]\exp(-\tfrac{\tau}{T}\ln\Delta_1), \\
\vec{v}_4(t) &= \left[\sin\left(-\Gamma_1\tfrac{\tau}{T}\right)e_3(\tau)+\cos\left(-\Gamma_1\tfrac{\tau}{T}\right)e_4(\tau)\right]\exp(-\tfrac{\tau}{T}\ln\Delta_1), \\
\vec{v}_5(t) &= \left[\cos\left(-\Gamma_2\tfrac{\tau}{T}\right)e_5(\tau)-\sin\left(-\Gamma_2\tfrac{\tau}{T}\right)e_6(\tau)\right]\exp(-\tfrac{\tau}{T}\ln\Delta_2), \\
\vec{v}_6(t) &= \left[\sin\left(-\Gamma_2\tfrac{\tau}{T}\right)e_5(\tau)+\cos\left(-\Gamma_2\tfrac{\tau}{T}\right)e_6(\tau)\right]\exp(-\tfrac{\tau}{T}\ln\Delta_2),
\end{aligned}$$

where $\Gamma_{1,2}=\arg(\lambda_{3,5})$.

Finding $\alpha_{new}, \delta_{new}$

The computation of variational equations (of suitable order) w.r.t. α and δ gives explicit expressions for the effect of different orientations (close to the reference values $\alpha = \alpha_0, \delta = \delta_0$) trajectory.

$$\phi_t(x_0, \alpha_0 + h_a, \delta_0 + h_d) = \phi_t(x_0, \alpha_0, \delta_0) + \frac{\partial \phi}{\partial \alpha}(x_0, \alpha_0, \delta_0) \cdot h_a + \frac{\partial \phi}{\partial \delta}(x_0, \alpha_0, \delta_0) \cdot h_d,$$

We can impose conditions on the "future" of the orbit and find orientations that fulfil them (or show that the condition is unattainable).

- For each mission we will define the parameters ε_{max}, Dt_{min} and Dt_{max} that can vary for each mission application.
- We will find α_{new}, δ_{new} and $dt \in [Dt_{min}, Dt_{max}]$ so that the trajectory comes back close to the nominal orbit.

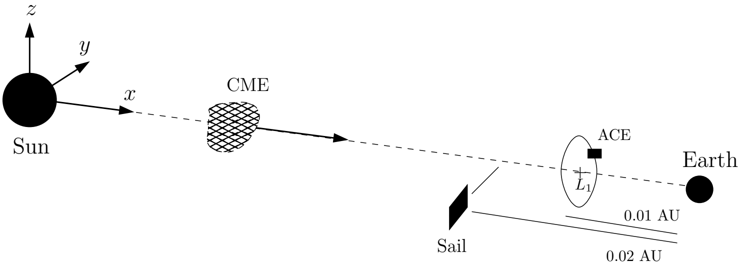
Finding $\alpha_{new}, \delta_{new}$

One can think of different ways to solve this problem. We have seen that the best results are found if we:

$$\phi_t(x_0, \alpha_0 + h_a, \delta_0 + h_d) = \phi_t(x_0, \alpha_0, \delta_0) + \frac{\partial \phi}{\partial \alpha}(x_0, \alpha_0, \delta_0) \cdot h_a + \frac{\partial \phi}{\partial \delta}(x_0, \alpha_0, \delta_0) \cdot h_d,$$

- For each $dt \in [Dt_{min}, Dt_{max}]$ we will find α_{new} and δ_{new} such that $s_1 = 0$ and (s_5, s_6) are minimum (i.e. we are close to stable direction and one of the centres is small).
- From all the dt, α_{new} and δ_{new} we chose the one such that the other centre projection (s_3, s_4) is minimised.

Results for the Geostorm Mission

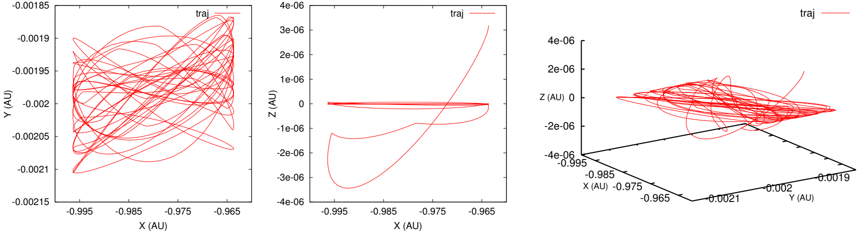


Mission Parametres:

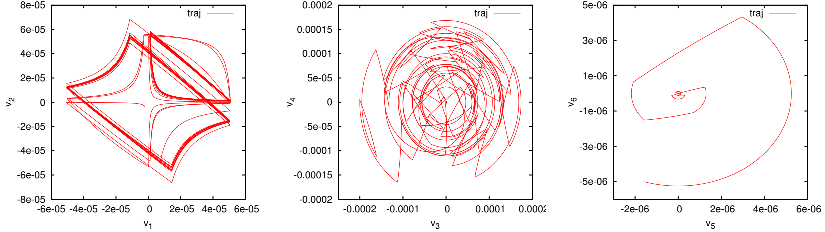
- In the RTBPS to have the appropriate fixed point we take: $\beta = 0.051689$ ($a_0 \approx 0.3$ mm/s^2), $\alpha_0 = 0.7897^\circ$ and $\delta_0 = 0^\circ$.
- We have taken $\varepsilon_{max} = 5 \cdot 10^{-5}$AU (the escape distance), $dt_{min} = 2$ days and $dt_{max} = 169$ days (the minimum and maximum time between manoeuvres).
- We have done simulations of the station keeping strategy up to 20 years.

Results for the Geostorm Mission

XY and XZ and XYZ Projections in a rotating reference system

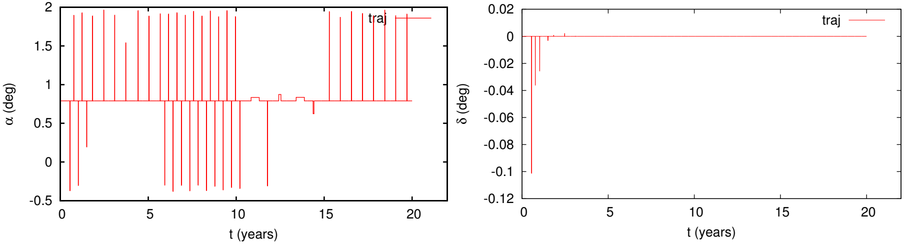


Saddle × Centre × Centre Projections



Results for the Geostorm Mission

Variation of the Sail Orientation



- In average we have a variation $\alpha \approx 1^\circ$ and $\delta \approx 0^\circ$.
- The trajectory takes about 100 days to escape.
- Two quick manoeuvres every 2 days are required each time we want to bring back the trajectory.

Conclusions & Future Work

Conclusion:

- We have described the dynamics of a solar sail in a Eath - Sun RTBP with a solar sail.
- We have derived station keeping strategies for a solar sail around an equilibrium point.
- We have extended these station keeping strategies to deal with a more more realistic model and applied them to the GeoStorm mission.
- Notice that these strategies do not require previous planning as the decisions are taken depending on the sails position at each time.

Future Work:

- Test the robustness of these strategies when different sources of error occur during the simulations (position and velocity determination or solar sail orientation).
- Include a more realistic model for the performance of the sail.

Gràcies per la vostra atenció