Invariant manifolds and transport in an Earth-Moon system perturbed by Sun's gravity field

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SIMBa

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Hamiltonian systems

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- Bicircular Problem

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- Dynamical substitute
- Invariant tori and stability
- Invariant manifolds

Transport through L_3 in the BCP

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- Change of coordinates
- Transport in a realistic model

Conclusions

Hamiltonian systems

- Newton's second law gives rise to systems of second-order differential equations in $\mathbb{R}^n$.
- That can be rewritten as a system of first-order differential equations in $\mathbb{R}^{2n}$.
- Being n an integer denoting the number of degrees of freedom of the system.

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A **Hamiltonian system** is a system of $2n$ first order ordinary differential equations of the form

$$\dot{q}_i = \frac{\partial H}{\partial p_i}(t, q, p), \qquad \dot{p}_i = -\frac{\partial H}{\partial q_i}(t, q, p), \qquad i = 1, ..., n, \tag{1}$$

where

- $H = H(t, q, p)$ is the Hamiltonian function, a smooth real-valued function defined for $(t, q, p) \in \mathcal{U}$, an open set in $\mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^n$.
- t denotes the time.
- $q = (q_1, ..., q_n)$ and $p = (p_1, ..., p_n)$ are the position and momentum vectors, respectively.
- Variables q and p are said to be conjugate variables.

Hamiltonian systems

System (1) can be reformulated in terms of the $2n$ vector $z = (q, p)$ and the $2n \times 2n$ skew symmetric matrix J and the gradient of the Hamiltonian function

$$\dot{z} = J \nabla H(t, z), \qquad J = \begin{pmatrix} 0 & \mathrm{I} \\ -\mathrm{I} & 0 \end{pmatrix}$$

where 0 is the $n \times n$ zero matrix and I is the $n \times n$ identity matrix.

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• If the Hamiltonian function is independent of time, $H = H(q, p)$, the differential equations are autonomous and the Hamiltonian system is called conservative.

• If the Hamiltonian function is dependent of time, $H = H(t, q, p)$, the differential equations are non-autonomous and the Hamiltonian system is not conservative.

Restricted Three-Body Problem (**RTBP**)

RTBP is an autonomous Hamiltonian system that describes the motion of an **infinitesimal particle** subjected to the **gravitational fields** created by two punctual massive bodies, called *primaries*, for us the Earth and the Moon, that are assumed to revolve in **circular motion** around their barycentre, where the origin is set.

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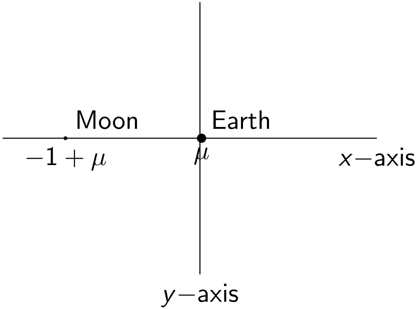
▷ In the **synodic** reference frame, the axis rotate with the primaries.



- Units are normalised, such that the gravitational constant is 1:
  - Length unit: Earth-Moon distance.
  - Mass unit: sum of the Earth and Moon masses.
  - Time unit: such that the Earth-Moon period is 2π.
- Earth, with mass $1-\mu$, is placed at $(\mu, 0)$.
- Moon, with mass μ, is placed at $(-1+\mu, 0)$.
- Being $\mu = 0.012150582$ the Earth-Moon **mass parameter**.

Restricted Three-Body Problem (**RTBP**)

The Hamiltonian function for the **planar Earth-Moon system** is:

$$H_{RTBP} = \frac{1}{2}(p_x^2 + p_y^2) + yp_x - xp_y - \frac{1-\mu}{r_{PE}} - \frac{\mu}{r_{PM}}$$

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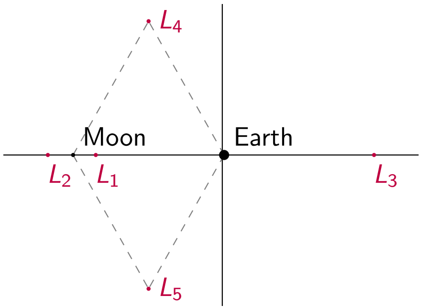

- Five equilibrium points, *Lagrangian points* L_j for $j = 1, \ldots, 5$, are found.
- Colinear points, L_1, L_2 and L_3, are unstable.
- Triangular points, L_4 and L_5, are linearly stable for the value of μ for the Earth-Moon system.

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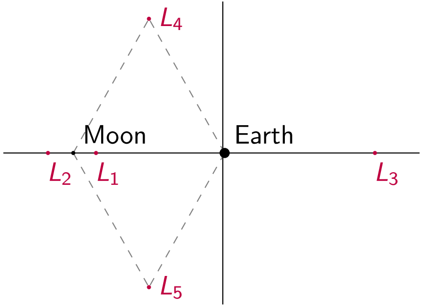

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- Colinear points, L_1, L_2 and L_3, are unstable.
- Triangular points, L_4 and L_5, are linearly stable for the value of μ for the Earth-Moon system.
- **We are interested in L_3.**

L_3 in the RTBP

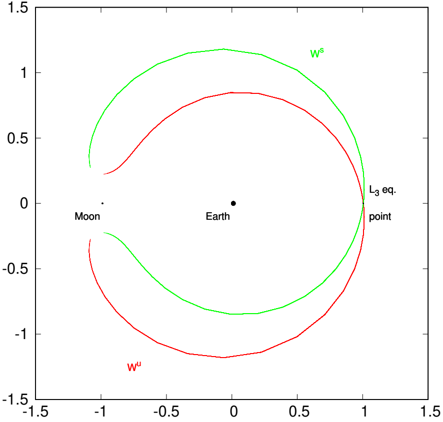
$\rightarrow$ L_3 equilibrium point is of centre $\times$ saddle type in the planar RTBP.

L_3 in the RTBP

→ L_3 equilibrium point is of centre × saddle type in the planar RTBP.

Due to the **SADDLE** part, we know that it has **stable** (W^s) and **unstable** (W^u) **invariant manifolds** associated:

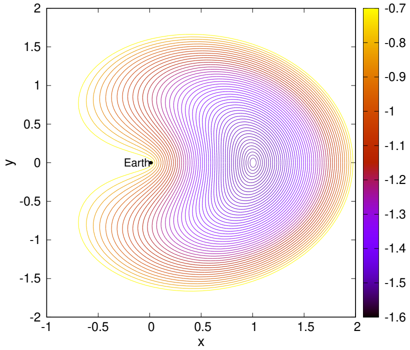
- W^s is composed by points that go **towards the eq. point** forward in time.
- W^u is composed by points that go **apart from the eq. point** forward in time.
- They are **bounded to an energy level**.



L_3 in the RTBP

According to the **Lyapunov Centre Theorem**, there exists a one-parametric family of **periodic orbits** emanating from L_3 equilibrium point in the **CENTRE** direction.

- Each periodic orbit is also partially **hyperbolic**; it has stable and unstable invariant manifolds associated.
- Each periodic orbit and its associated manifolds are **bounded to an energy level**.



Coloured according to their energy level.

Bicircular Problem (**BCP**)

For an accurate analysis of L_3 in the Earth-Moon system, it is necessary to introduce the gravitational effect of the Sun.

$\rightarrow$ A simple way of introducing this effect is through the **Bicircular Problem**, a modification of the RTBP, that describes a **restricted 4-body problem**.

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Assumptions about the **third massive body** (Sun):

- To be contained in the **same plane of motion** than the primaries (Earth and Moon).
- To revolve in **circular motion** around the original set up of the RTBP.
- To affect the **motion of the particle** but not the primaries.

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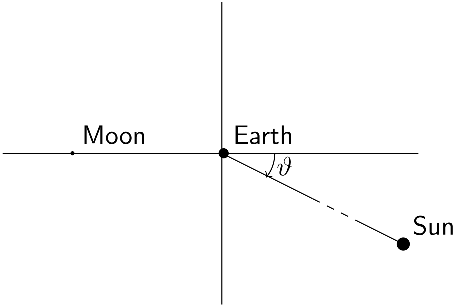
The third massive body acts as a **time-periodic perturbation of the RTBP**.

Bicircular Problem (**BCP**)



- **Non-autonomous Hamiltonian** system, then energy is not conserved.
- $\vartheta = \omega_s t$, with ω_s being the angular velocity of the Sun, denotes the **angular position of the Sun** respect to the Earth-Moon system

$$H_{BCP}(t) = H_{RTBP} + \hat{H}_{BCP}(t), \qquad \hat{H}_{BCP}(t) = -\frac{m_s}{r_{PS}} - \frac{m_s}{a_s^2}(y \sin(\omega_s t) - x \cos(\omega_s t))$$

being m_s the mass of the Sun and r_{PS} and a_s the distances from the Sun to the particle and to the Earth-Moon barycentre.

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BCP is said to inherit the dynamics of the RTBP.

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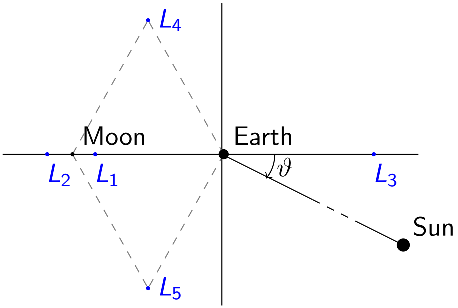
- This is one of the main questions to which **KAM theory** is devoted.
- From the works of Kolmogorov, Arnold and Moser, it is concluded that **most of the invariant solutions survive** under the perturbation, increasing their angular dimension.
- The reason why some of the solutions do not survive is due to **resonances** between the basic frequency vector of the system and the frequency introduced by the perturbation.

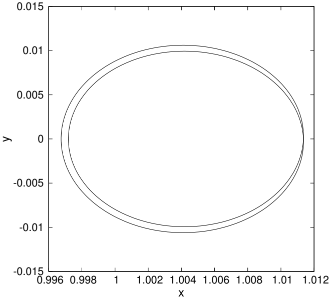
Bicircular Problem (**BCP**)



- The five equilibrium points L_j for $j = 1, ..., 5$ are replaced by periodic orbits with the **period of the perturbation** ($T = \frac{2\pi}{\omega_s}$).

Dynamical substitute for L_3 in the BCP

The dynamical substitute for L_3 in the BCP is **a periodic orbit of period** T, whose stability is again of centre $\times$ saddle type.



Then

- It has **stable** (W^s) and **unstable** (W^u) **invariant manifolds** associated.
- In the centre direction, there emanates a one-parametric family of two-dimensional Lyapunov quasi-periodic solutions (**2D invariant tori**).

Invariant tori and stability

Family of 2D invariant tori around L_3 dynamical substitute

- A family of **quasi-periodic orbits** emerges in the centre direction from L_3 periodic orbit.
- Each of the tori composing this family has **two frequencies**:
  - one comes from the family of Lyapunov periodic orbits of L_3 in the unperturbed system and it is different for each torus,
  - the other one is the frequency of the Sun, shared by them all.

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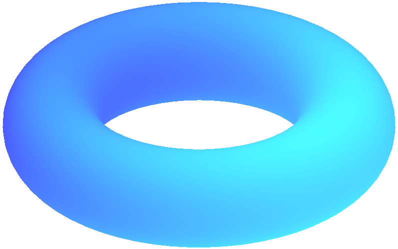
Temporal Poincaré map P

Stroboscopic map at time equal to the period of the Sun (T) is applied to the flow, **reducing one angular dimension**. In this map:

- The dynamical substitute is seen as a fixed point.
- The family of 2D invariant tori is seen as a family of 1D invariant curves.

Curves in the map temporal Poincaré map

A **two dimensional torus** in the flow.



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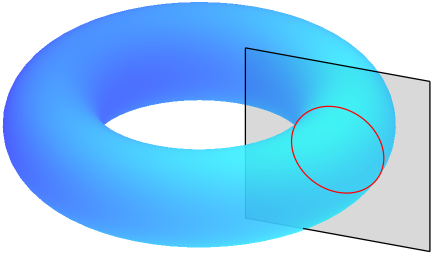

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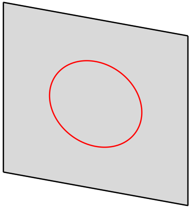

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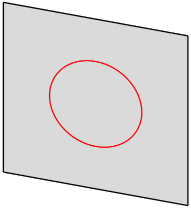
We study the 2D tori of the flow through the 1D curves in the map.



Invariant tori and stability

Family of 1D invariant curves around L_3 in the map P

- Each curve, $\varphi : \mathbb{T} \mapsto \mathbb{R}^{2n}$ with $n = 2$, is characterized by its **rotation number** ω.
- Each curve must satisfy **invariance condition**:

$$P(\varphi(\theta)) = \varphi(\theta + \omega), \qquad \theta \in \mathbb{T}.$$

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- For their **computation** we approximate each curve as a truncated **real Fourier series**:

$$\varphi(\theta) \approx \alpha_0 + \sum_{\kappa=1}^{N} \alpha_\kappa \cos(\kappa\theta) + \beta_\kappa \sin(\kappa\theta),$$

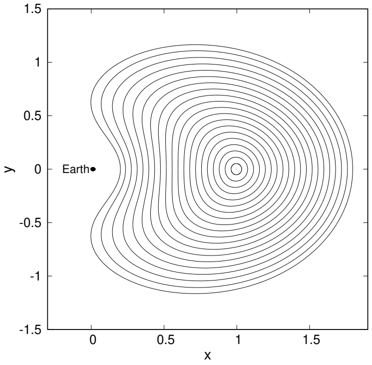
where α_0, α_κ, β_κ, are the Fourier coefficients with $\kappa = 1, ..., N$ and $\theta \in [0, 2\pi)$, and look for the invariance condition to be satisfied by means of a **Newton method**.

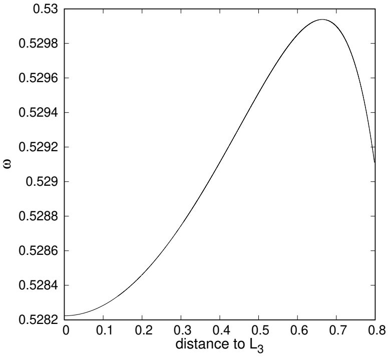
Invariant tori and stability

Family of 1D invariant curves around L_3 in the map P

- Each curve φ with rotation number ω, satisfies invariance condition:

$$P(\varphi(\theta)) = \varphi(\theta + \omega).$$





Invariant tori and stability

Linear behaviour around a quasi-periodic solution φ

- It is described by the **linear quasi-periodic skew-product**

$$\begin{cases} \bar{\varphi} = A(\theta)\varphi, \\ \bar{\theta} = \theta + \omega. \end{cases}$$

where $A(\theta) = D_\varphi(P(\varphi(\theta)))$ is the Jacobian of the Poincaré map on φ.

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- We look for pairs of eigenvalue and eigenfunction (λ, ψ) that satisfy the **generalized eigenvalue problem** (GEV),

$$A(\theta)\psi(\theta) = \lambda T_\omega \psi(\theta),$$

  where T_ω is the operator $T_\omega : \psi(\theta) \in C(\mathbb{T}, \mathbb{C}^4) \mapsto \psi(\theta + \omega) \in C(\mathbb{T}, \mathbb{C}^4)$.

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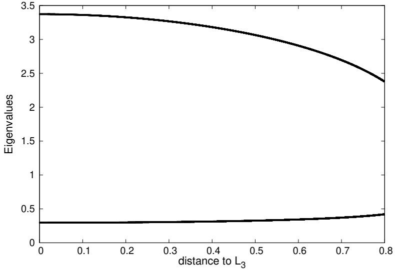
- We solve this system also in terms of **Fourier series** and by means of **Newton methods**.

Invariant tori and stability

Linear behaviour around a quasi-periodic orbit φ

These 2D invariant tori are partially **hyperbolic**, with eigenvalues:



- Stable eigenvalue $\lambda_s < 1$.
- Unstable eigenvalue $\lambda_u > 1$.
- $\lambda_u = \lambda_s^{-1}$ due to the **Hamiltonian structure**.

These 2D tori have **stable** (W^s) and **unstable** (W^u) **invariant manifolds** associated.

Invariant manifolds of invariant tori

- The **stable** (W^s) and **unstable** (W^u) **invariant manifolds** associated with two dimensional quasi-periodic orbits, are **three dimensional in the flow**.

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Stable/Unstable invariant manifolds in P can be parametrised with two parameters.

- The angle $\theta \in \mathbb{T}$ along the invariant curve.
- A parameter $\sigma \in \mathbb{R}$.

Invariant manifolds of invariant tori

Linear approximation of invariant manifolds

We take an small displacement ($\sigma \in \mathbb{R}$) in the hyperbolic (stable or unstable) direction:

$$P(\varphi(\theta) + \sigma\psi_{s,u}(\theta)) = P(\varphi(\theta)) + \sigma D_{\varphi}(P(\varphi(\theta)))\psi_{s,u}(\theta) + (\sigma^2)$$

$$= \varphi(\theta + \omega) + \sigma\lambda_{s,u}\psi_{s,u}(\theta + \omega) + (\sigma^2).$$

- The displacement must be taken in positive ($\sigma > 0$) and negative ($\sigma < 0$) values.

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- $\sigma \in [\sigma_0, \sigma_0\lambda_u]$ (or $\sigma \in [\sigma_0, \sigma_0/\lambda_s]$) and $\theta \in \mathbb{T}$ so that

$$(\theta, \sigma) \mapsto \varphi(\theta) + \sigma\psi_{s,u}(\theta)$$

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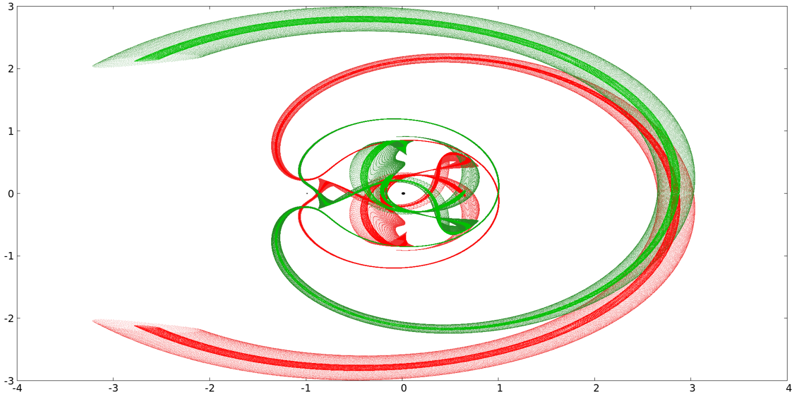
  parametrises a cylinder-shaped fundamental domain on the invariant manifold.
- **Unstable manifolds** are propagated forward in time and **stable** backward.

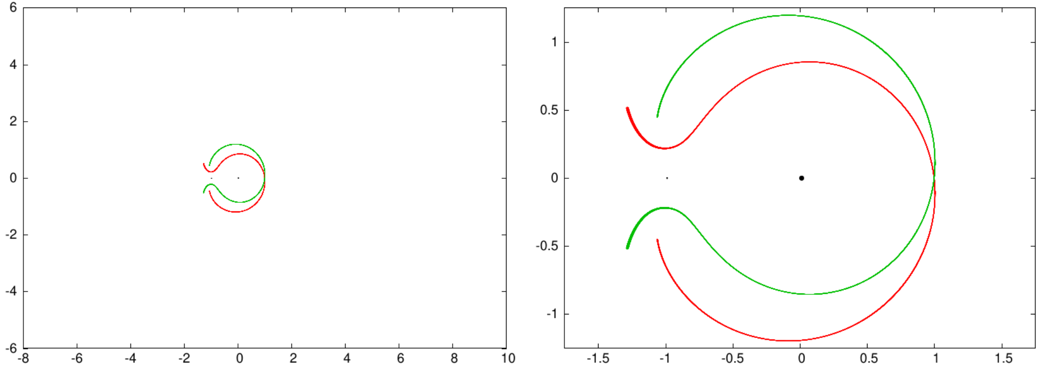
Transport through L_3 in the BCP

At every step of the integration we check if the orbits collide with some primary or if they leave the system (defined as being further than 10 Earth-Moon distances from their barycenter).



Stable (green) and unstable (red) invariant manifolds corresponding to two invariant curves, in the *xy*-plane.

Transport through L_3 in the BCP



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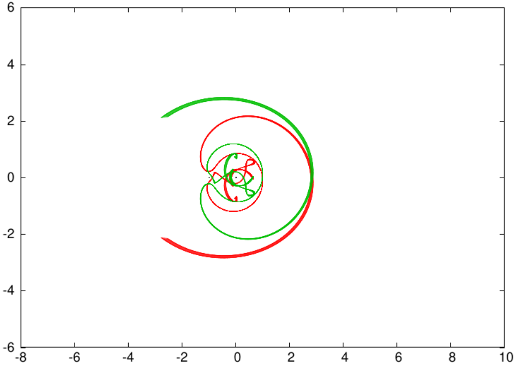

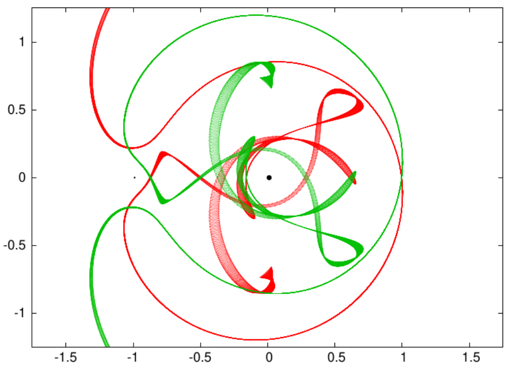

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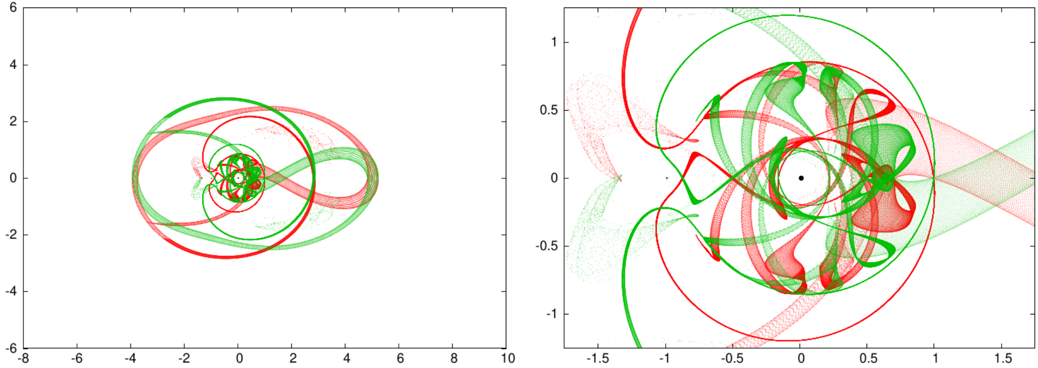

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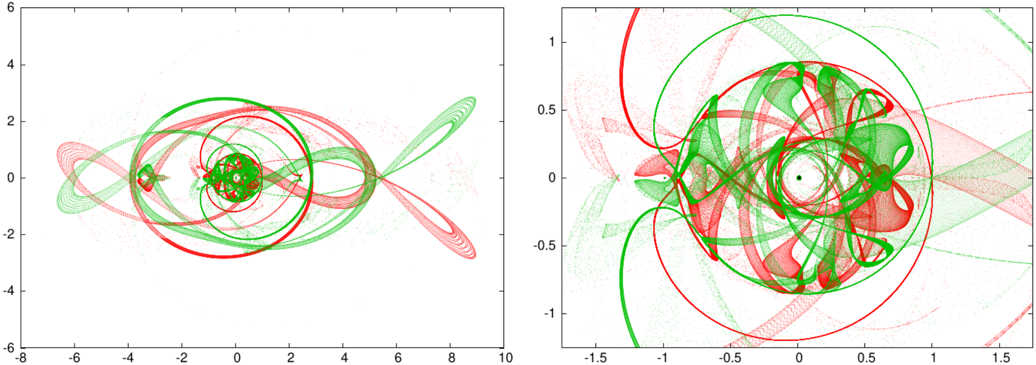


Transport through L_3 in the BCP

Fundamental cylinder

Fundamental domain (the small "cylinder") used for globalising the invariant manifolds, is defined by two parameters (θ, σ).

For example, the parametrization of the fundamental region of the **unstable manifold** for an invariant curve φ is perfomed as:

$$(\theta, \sigma) \in [0, 2\pi] \times [\sigma_0, \lambda_u \sigma_0] \mapsto \varphi(\theta) + \sigma \psi_u(\theta),$$

for $\sigma_0 > 0$ and $\sigma_0 < 0$.

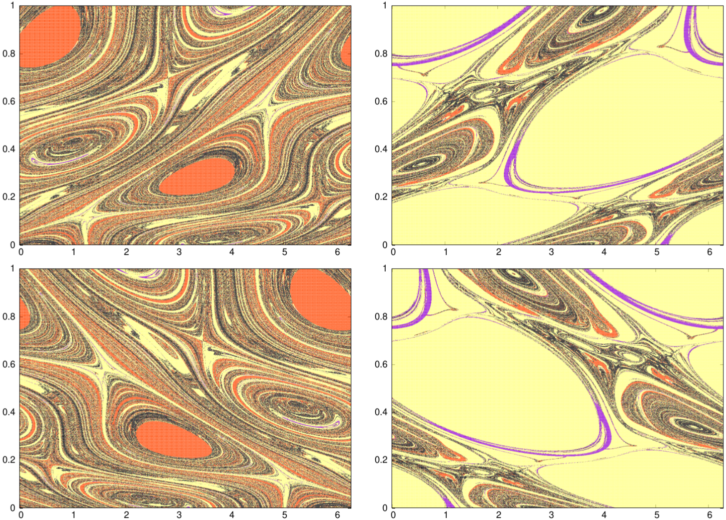
With these two parameters we define a **mesh of initial points** of the four invariant manifolds for an invariant curve and **colored them according to their fate**.

Transport through L_3 in the BCP

Color (fate): Purple (Earth), red (Moon), yellow (leaving the system), or black (neither).
Invariant torus at 0.03335 from L_3.

Unstable manifold, Left/right, taking positive/negative displacement.

Stable manifold, Left/right, taking positive/negative displacement.



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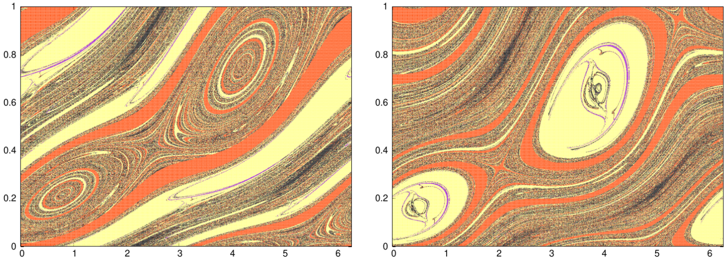
Unstable manifolds of invariant tori at **0.19607** from L_3.



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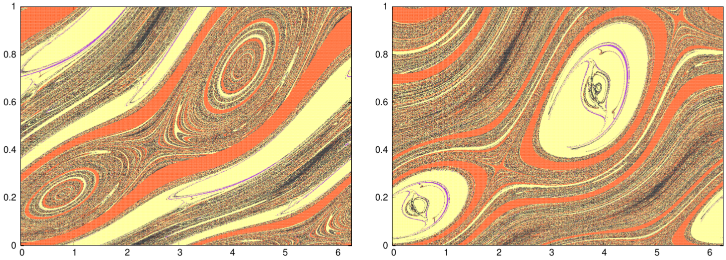
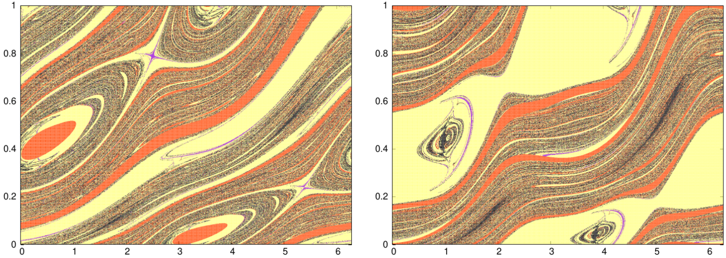


Unstable manifolds of invariant tori at **0.30902** from L_3.



Transport through L_3 in the BCP

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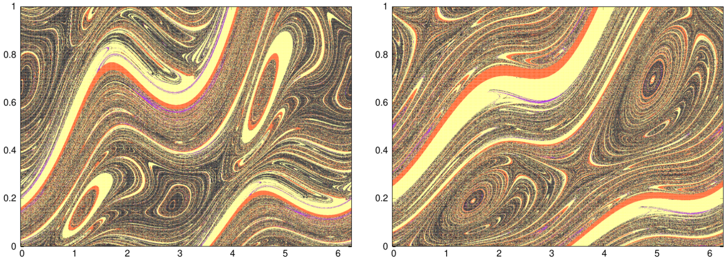
Unstable manifolds of invariant tori at **0.57020** from L_3.



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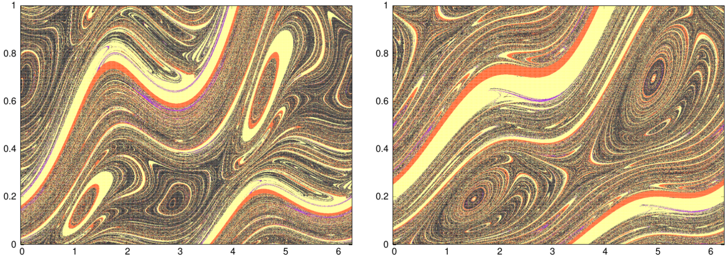
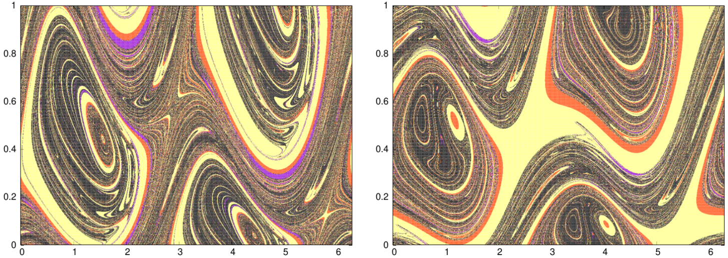
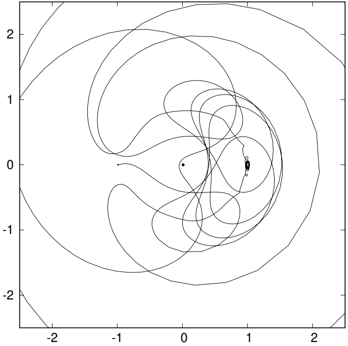


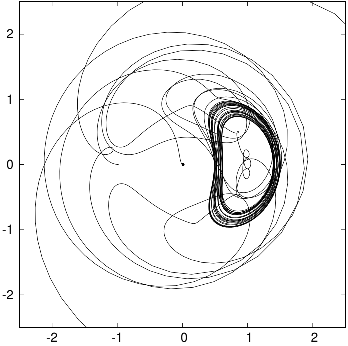
Unstable manifolds of invariant tori at **0.74214** from L_3.



On the existence of heteroclinic orbits

A phenomenom observed thanks to the not conservation of the energy: some orbits suggest the existence of intersections between the manifolds of different invariant curves near L_3.





Left, an orbit that goes from the Moon surface to the outside system through (*a priori*) an inner torus. Right, an orbit that goes from the Moon to the Earth through (*a priori*) an outer torus.

Lunar meteorites

▷ It is known that:

- Moon surface suffers several **impacts** every year.
- If the velocity of the crater ejecta is higher than the lunar escape velocity (≈ 2.38 km/s), they get free from the Moon gravity and become **lunar meteorites**.
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In fact, several of these connections have been found for the BCP.

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  - **Modify** their initial **velocity modules** and **angle directions** of the velocity vector, such that a mesh of 10^6 initial conditions is swept.
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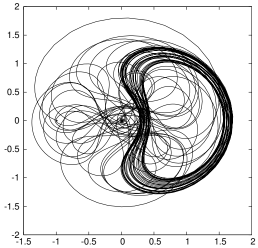
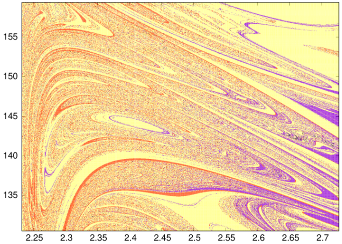


xy-plane (adim units)



|v| (km/s), angle dir. (degrees)

Color	(**fate**):
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Time: In the BCP at $t = 0$ or $t = N_T T$ ($N_T \in \mathbb{Z}$), the positions of the Earth, the Moon and the Sun correspond to a **lunar eclipse**, $T_{ECLIPSE}$ in Julian days.

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Coordinates: The conversion to the **ecliptical system** with the origin in the Solar System centre of mass involves the coordinates of Earth, Moon and their barycentre **at that real time**.

$\rightarrow$ we take the coordinates of Earth, Moon and their barycentre from **JPL database** (Jet Propulsion Laboratory).

Change of coordinates

Let R_E (V_E) and R_M (V_M) be the positions (velocities) of the Earth and the Moon, taken from the Solar System center of mass.

The relation between the position of a particle in the **adimensional system** (a) with the origin at the Earth-Moon barycentre and its position in the **ecliptical system** (e) with the origin at the Solar System c.o.m., is given by:

$$e = kCa + b,$$

where:

- $k = ||R_E - R_M||$ is the change of **scale factor**.
- C is a **rotation matrix** that depends on R_E, R_M, V_E and V_M.
- b is **Earth-Moon barycenter** taken from Solar System center of mass.

Lunar meteorites

Objective: to check the results obtained with the Bicircular model for the lunar meteorites in a more realistic model.

- Apply the **change of coordinates and time to each initial condition** in our adimensional system to translate them to the ecliptic system.

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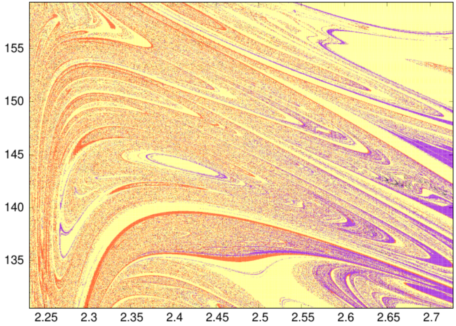
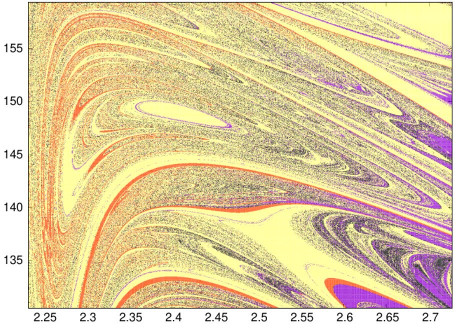
BCP



JPL



Horizontal axis: $|v|$ (km/s). Vertical axis: angle dir. (degrees).

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  - Use them to analyse the **capture of an asteroid** in the BCP.

Thank you for your attention!